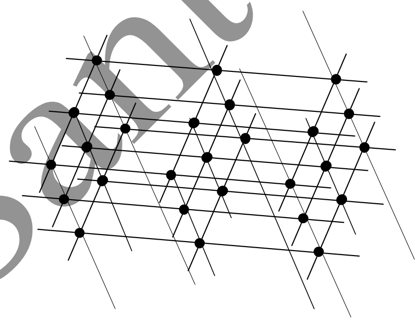
Organization of Lattices by crystal systems

Introduction

In last chapter, we have seen that the rotational symmetry along with translation symmetry leads to only **five types of two dimensional Lattice** (called NET). When we stack these nets one above other, it leads to three dimensional lattice, called **Bravais lattice.** Now obvious question would be how many such independent permutation and combinations are possible? In other words, how many such Bravais lattices possible with five different types of nets? Soon we will we will realize that there are only 14 different independent combination; so called 14 Bravais lattice which can be classified into only seven independent crystal systems. We start now investigating in an order.

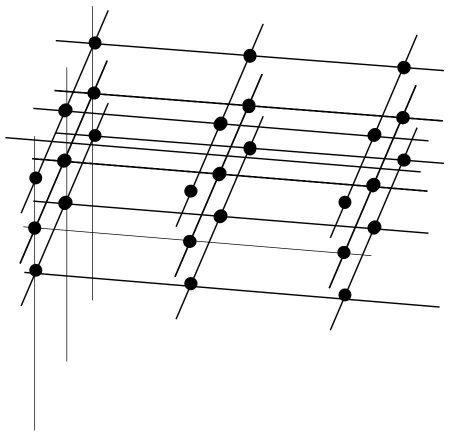
Triclinic System:

As we have mentioned above we are going to stack one net above other to get Bravais



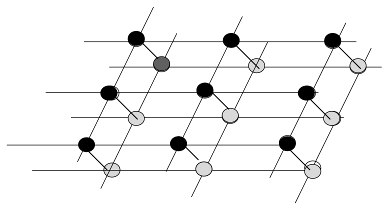
lattices. We will start stacking systematically from most asymmetric net to most symmetric one. We will start with stacking **Oblique Nets** one above other. While stacking we have obvious two choices (1) Coinciding the lattice points of two nets i.e. putting the vector '**c**' perpendicular to the plane in which vector **a and b** is lying. (2) or giving little offset so that vector **c** is not perpendicular to the plane **ab.**

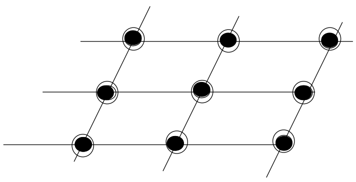
If we stagger the next plane with little offset so that these axes do not coincides, then the C_2



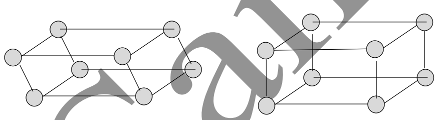
axis symmetry of obliged net will break. The Bravais lattice so produced is called Triclinic, the only member of ***triclinic system.***

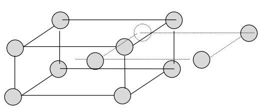
If we try to preserve the two fold symmetry? Then what we should do ? where you should put second net ? Here there are two



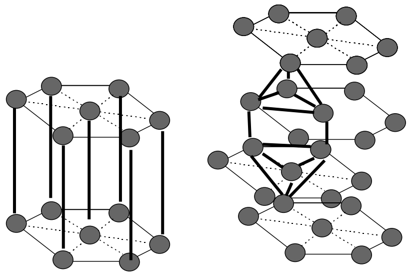


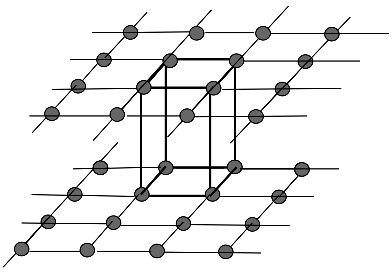
possibilities. (1) we can achieve this by either putting second net directly coinciding with first net as we have discussed in the last section where vector **c** is perpendicular to plane **ab** the system what we get is **monoclinic. The unit cell what we get is called primitive (P) monoclinic** (Why primitive? Because in chosen unit cell the same kind of basis are lying at the eight corners only) (2) We can still retain the two fold symmetry by placing the second net at the center of parallelogram. This leads to **body centered monoclinic.**

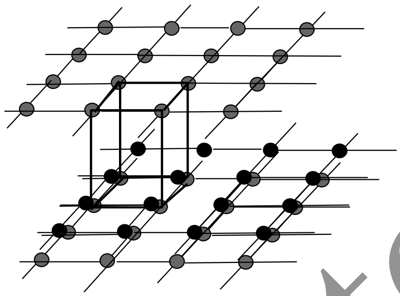




From Two fold symmetry we now move to more symmetric net. Therefore, in this series the next member is a net with **three fold axis symmetry i.e. hexagonal net** which has three fold axis symmetry through triangle centers and six fold axis symmetry through lattice point. At present, we are only interested in three fold axis symmetry. Therefore, we will put next hexagonal net such that lattice point of the one net will be lying at the center of the triangle. The resultant arrangement is called **Rhombohedral lattice.** It is only member of a triagonal system. We will move now to more symmetric object. We are now Moving from three fold axis symmetry to four fold. Obviously we have to make use of **square net**. Again we have two choices (as in the case of monoclinic) results into **primitive tetragonal and body centered tetragonal**







From four fold symmetry we will now go to six fold symmetry. From the continuation of a discussion from triagonal system, if we start with hexagonal net stacked such that they are having coincide lattice points it leads to **hexagonal lattice.** Obviously we can not arrange it by any other way so that it will mention the six fold axis symmetry and only one type of cell.

If we start with rectangular net, How many ways we can stack the rectangular net so that we can mention two ‘two fold symmetry axis’ perpendicular to each other ? Here we can exploit all the symmetry axis centers on the net these are (1) lattice points, that leads to primitive one. (2) both the sides which leads

HOME ASSIGNMENT:

Determine the symmetry group of all seven crystal systems. Express the groups in the international notation also.

to **base centered or side centered** (3) and center of the rectangular which leads to body centered. Starting from rhombic net one can show how the **face centered orthorhombic system can be built.**

Exactly under same fashion, we can arrange square net i.e. having four fold axis into three different Bravais lattice so that ultimate emerged structure will mention the four fold axis symmetry. These are **simple cubic, body centered cubic and face centered cubic.** We can now list all fourteen Bravais lattice which are organized in seven crystal systems. It is

System	**Symbols of conventional Unit cell**	**Axial relationships with conventional unit cells**	**Symmetry of each lattice points**
Triclinic	P	$a \neq b \neq c$, $\alpha \neq \beta \neq \gamma \neq 90 or\ 120$	$\bar{1}$ (i)
monoclinic	P,C	$a \neq b \neq c$, $\alpha=\gamma=90^o$ and $\beta \neq 90$ or 120	2/m, (C_{2h})
Orthorhombic	P,C,I,F	$a \neq b \neq c$, $\alpha=\beta=\gamma=90^o$	mmm , (D_{2h})
Tetragonal	P,I	$a = b \neq c$, $\alpha=\beta=\gamma=90^o$	$\frac{4}{m}mm$ (D_{4h})
Cubic	P,I.F	a=b=c, $\alpha=\beta=\gamma=90^o$	m3m (O_h)
Hexagonal	P	$a = b \neq c$, $\alpha=\beta=90^o$,γ=120	$\frac{6}{m}mm$,(D_{6h})
Trigonal	R	a=b=c, $\alpha=\beta=\gamma \neq 90, <120$	$\bar{3}$ m (D_{3d})

as follows.