Rotational symmetry of Bravais lattice:

A rotational axis of a Bravais lattice is a line passing through lattice point, and lattice remains indistiuishable after rotation about some specific angle. If the axis is translated with action of translation vector, it is clearly still is a rotation axis. The Two such axes are said to be equivalent. Since a lattice consists of desecrate points, rotation at any angle will obviously not reproduce the original structure; there must be finite desecrate angles φ_o across which if lattice rotated, the original structure is reproduced. Any other angle φ which represent symmetry rotation about same axis has to be integral multiple of φ_o

$\frac{\varphi}{\varphi_o} \leq \nu < \frac{\varphi}{\varphi_o} + 1$ where ν is uniquely defined integer which also satisfies the condition, $0 \leq \nu\varphi - \varphi_o < \varphi_o$ but since φ and φ_0 both are allowed rotation ($\nu\varphi_0$- φ) is also allowed.

Obviously for every rotation axis has 360^o is an allowed rotational angle it follows that φ_0 has to be integral multiple of 360^0 and (360/ φ_0)=n *number of folds of symmetry axis has.*

Inseparable[1] translation symmetry from rotation symmetry constrains the value of 'n' equal to 2, 3, 4, and 6.

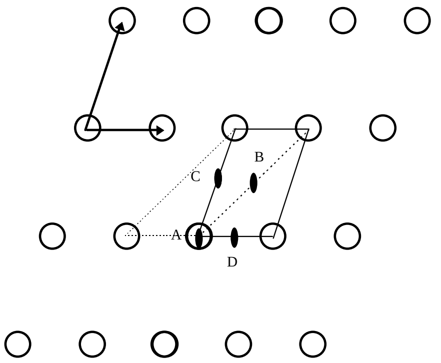
Also the plane perpendicular to the rotational axis and containing one lattice point contains infinite numbers of two dimensional net of points. Therefore, the whole lattice can be generated by stacking such nets on top of each other.

Before creating such 3D lattices by stacking nets, we must first determine the rotation symmetry of each such two dimensional net.

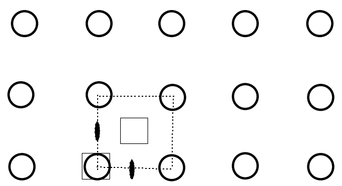
[1] (the put its explanation as an assignment)

Plane nets:

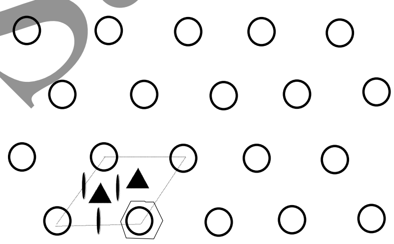
How do you create the net[2] of ***least*** *possible symmetry?*



It can be created by two primitive vectors whose length and direction are unrelated to each other. Such net is called '**oblique'** as shown in the figure. The obligue net already has four nonequivalent two fold axes point. One through lattice point 'A'. One through parallelogram center 'B'. Two through center of the parallelogram edges (C and D). The distinction among them will only depend on choice of the unit cell. eg. If we choose, another parallelogram as a unit cell (as shown by dotted line). The two fold axis point 'C' will now become center of the parallelogram rather than 'B' as a center. Therefore, it can be said that the oblique net has two types of axes, one goes through lattice point and another do not. In conclusion, the if the net has 'n' axis passing through lattice points there must exist axis passing anywhere other than the lattice points.

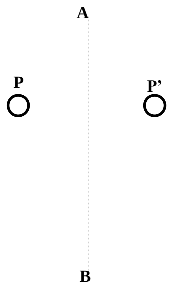


In the oblique lattice, all four axes has two fold symmetry (A,B,C and D). If two of the two fold axis becomes four folds it leads to *square net*.

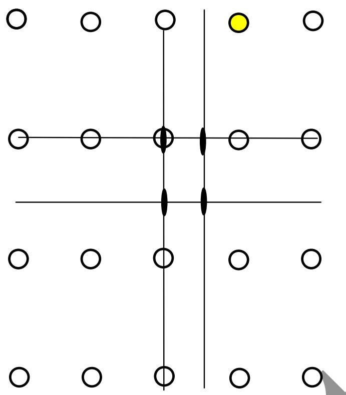


If one of the oblique net two fold axes is converted to six-fold it leads to hexagonal net. In this in addition to, two fold axes two three fold axes also appears. This completes the classification of nets by perpendicular rotation axis.

[2] Two dimensional lattice or 'net'work is called **net.**



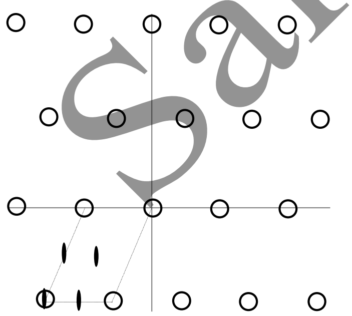
In general the axis which is not perpendicular to the plane of net can no be used as a symmetry axis. But there exists important exception about the axes which are lying in the plane itself which acts as reflection line with in the net. Let 'AB' be the reflection line, and 'P' be the lattice point in the plane. The reflection operator then produces another point P' indicates that there exists lattice translation vector perpendicular to AB. Then it is possible to choose primitive translation vector $\mathbf{a_1}$ perpendicular to AB. Such a line perpendicular to AB has *two nonequivalent reflection points.* (1) The lattice point it self and (2) midway between two lattice points.



There are two oblique nets which depicts reflection symmetry

(1) rectangular net which has along with four nonequivalent rotation axes, there exists four nonequivalent reflection lines.

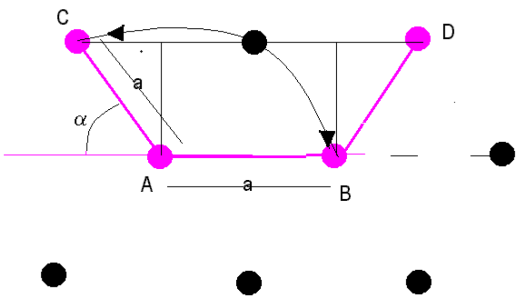
(2) Rhombic net, which has two nonequivalent reflection lines along with four nonequivalent rotation axes.



From the above discussion it is cleared that only 2,4 and 6 fold rotational symmetry only can go along with the translation operation. This can very easily be proved on the basis of trignometical argument.

Let us point A and B in the figure be the sites of some plane lattice.

Such that |A|= a (lattice const)



Let 'n' fold axes passing perpendicular to the lattice plane . supose one such axis passing throuh "A" . If we carry out rotational symmetry operation to get identical structure, we have to rotate it by angle 'α'= 360/n . After doing so we will get point 'C' and 'D'. The presence of translational symmetry requires that the new point 'C' and 'D' should coinside with the lattice point. This requirement can be express as $|CD| = m \times a$, where 'm' is an integer. From simple geometrical considaration, we have $m.a = a + 2\times a. \cos(\alpha)$ where, $m = 0, \pm1, \pm2, \ldots$.

$\pm$ is used because the rotation is either clockwise or antoclockwise. Dividing both the side by 'a' we get,

$m = 1+2\cos(\alpha)$ or $(m-1)=2\cos(\alpha)$

since 'm' is an integer, (m-1) will also be an integer. Take (m-1)= N

then, $N= 2\cos(\alpha)$ or ***$\cos(\alpha)=N/2$*** where $N = 0, \pm1, \pm2, \ldots$.

As all the values of $\cos(\alpha)$ lies in the range 1 to –1 corresponding values of N, α, and n can easily obtained.

N	**cosα**	**α**	**'n' allowed rotations**
-2	-1	180	2
-1	-1/2	120	3
0	0	90	4
+1	½	60	6
+2	+1	360 or 0	1

This table clearly shows the non existing of symmetry axes other than 1,2,3,4,6