Assignment Number 5 SKH CD100

Tuesday, October 23, 2007

1. Describe the following wave functions as symmetric (even), antisymmetric (odd) or neither (unsymmetric or asymmetric):
   a) $\Psi(\theta) = \cos\theta$ b) $\Psi(\theta) = \sin\theta\cos\theta$ (c) $\Psi(x) = A\exp(-x)$, where A is a constant(d) $\Psi(x) = x^n$, where n is odd; and (e) $\Psi(x) = x + x^2$
2. Identify which of the following wave functions are "acceptable":
   a) $\Psi(x) = \pm x^2$ b) $\Psi(x) = Ax^2$ where A is a constant (c) $\Psi(\theta) = \cos\theta$
   (d) $\Psi(x) = A\exp(-ax)$, where a is a constant
3. Determine $\Psi^*\Psi$ for the following wave functions
   a) $\Psi(\theta) = \sin\theta + i\cos\theta$ b) $\Psi(x) = A\exp(iax)$ and (c) $\Psi(x) = \exp(-x^2)$, where $i = (-1)^{\frac{1}{2}}$
4. For the wave function $\Psi(\theta) = A\exp(im\phi)$ where m is an integer, evalvate A so that the wave function is normalized?
5. Show that the wave functions $\Psi_1(x) = \sin\left(\frac{n\pi x}{a}\right)$ and $\Psi_2(x) = \cos\left(\frac{n\pi x}{a}\right)$, where *n* and *a* are constants, are orthogonal. The permitted values of *x* are $0 \le x \le a$.
6. Show that the wave functions $\Psi_1(\phi) = A\exp(im\phi)$ and $\Psi_2(\phi) = B\exp(im\phi)$, where *m* and *n* are constants, are orthonormal.
7. The operators for position and linear momentum are given by $\hat{x} = x$ and $\hat{p}_{(x)} = \frac{\eta}{i}\frac{\partial}{\partial x}$ respectively. Determine the result of operating on the function $\Psi(x) = A\sin\left(\frac{n\pi x}{a}\right)$, where A, n and a are constants, with each operator.
8. Show that the wave functions describing a 1s electron and 2s electron are orthogonal.

$$\Psi_{1s} = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{\pi^{\frac{1}{2}}} \exp(-\frac{\rho}{2}) = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{\pi^{\frac{1}{2}}} \exp(-\frac{Zr}{a_0})$$

$$\Psi_{2s} = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{4(2\pi)^{\frac{1}{2}}} (2-\rho)\exp(-\frac{\rho}{2}) = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{4\pi^{\frac{1}{2}}} (2-\frac{Zr}{a_0}) \exp(-\frac{Zr}{2a_0})$$